II.—HEGEL'S TREATMENT OF THE CATEGORIES OF QUANTITY.

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In this paper, as in my previous papers on the details of Hegel's Logic (Mind, April and July, 1897; January, 1899; April, 1900; October, 1902), I shall consider one of the nine secondary divisions of the process. I shall follow, in the first place, the exposition in the Greater Logic, and add a few words on the rather different treatment adopted in the Encyclopædia.

Quantity (*Quantität*) is the second division of the Doctrine of Being. It is divided as follows:—

I.—QUANTITY (QUANTITÄT).

A.—Pure Quantity (Die reine Quantität).

B.—Continuous and Discrete Magnitude (Kontinuirliche und Diskrete Grösse).

C.—Limitation of Quantity (Begrenzung der Quantität).

II.—QUANTUM (QUANTUM).

A.—Number (Die Zahl).

B.—Extensive and Intensive Quantum (Extensives und Intensives Quantum).

(*a*) *Their Difference* (*Unterschied derselben*).

(*b*) *Identity of Extensive and Intensive Magnitude* (*Identität der Extensiven und Intensiven Grösse*).

(*c*) *The Alteration of Quantum* (*Die Veränderung des Quantums*).

C.—THE QUANTITATIVE INFINITY (DIE QUANTITATIVE UNENDLICHKEIT).

(*a*) *Its Notion* (*Begriff derselben*).

(*b*) *The Quantitative Infinite Progress* (*Der Quantitative Unendliche Progress*).

(*c*) *The Infinity of Quantum* (*Die Unendlichkeit des Quantums*).

III.—THE QUANTITATIVE RATIO (DAS QUANTITATIVE VERHÄLTNISS).

A.—THE DIRECT RATIO (DAS DIREKTE VERHÄLTNISS).

B.—THE INVERSE RATIO (DAS UMGEKEHRTE VERHÄLTNISS).

C.—THE RATIO OF POWERS (POTENZENVERHÄLTNISS).

It will be noticed that Quantity is used in an ambiguous sense here, since it is the name both of the whole secondary division, and of the first of the tertiary divisions contained in it. The tertiary division might be distinguished if we gave it the name of Indefinite Quantity, which, as we shall see, would be appropriate to it.

The treatment of Quantity is not one of the most successful parts of the dialectic. Hegel devotes a larger proportion of the Greater Logic to it than he does to any of the eight other divisions. Yet the transitions are frequently obscure, and often appear to owe their obscurity to excessive compression. By far the greater part of the 185 pages which are employed on Quantity are occupied with notes on collateral points. Some of these, indeed, throw additional light on the main argument, but the rest only contain criticisms of Kant's views on Quantity, and of certain mathematical doctrines. Hegel is never at his best when criticising Kant, and this is eminently the case here. The mathematical discussions, again, are too purely technical to give us much assistance in comprehending the course of the dialectic. Moreover, it may possibly be said that on this occasion, as on some others, Hegel yielded to the temptation of criticising a science whose contents were not adequately known to him.

It is easy, however, to exaggerate the effect of such faults in destroying the value of this part of the dialectic. The transitions, though in some cases obscure, can, as I shall

endeavour to show, be understood. And if they are valid, the mathematical mistakes, if such there are, are of small importance. The main object of the dialectic, after all, is to reach the Absolute Idea, and so to demonstrate what is the true nature of reality. Thus the principal function of the lower categories is to lead on to the Absolute Idea. And for this it is only requisite that each of them should validly follow from the one which precedes it, and lead on to the one which follows it.

Now the question whether Hegel's various categories of Quantity do perform this function is not affected by any mathematical mistakes which he may have made, nor can it be settled in the negative by any mathematical criticisms. The only relevant inquiry is whether Hegel was justified in starting the dialectic with the category of Pure Being, and whether the validity of the various categories of Quantity can be shown to be involved in the validity of the category of Pure Being. And this inquiry is a matter for metaphysics, and not for mathematics.

It is true that Hegel's main aim in the dialectic was not his only aim. He wished, not merely to deduce an absolutely valid conception of reality, but to account for other less perfect conceptions, and to range them in the order of their relative validity. He probably believed that the categories with which he dealt in the sphere of Quantity were identical with the fundamental notions of mathematics. In so far as this is not the case, he must be considered to have failed in his subordinate purpose, and, in so far as he has failed, to have introduced additional obscurity by the fact that he has called his categories by the names of the mathematical notions.

But the purpose in which he may have failed is, as I have said, only of subordinate importance for him. And, even in that purpose, his failure would not be a sign of any metaphysical flaw in his system, but simply of mathematical ignorance. If the dialectic process is correct, it will be true of all mathematical conceptions, as of all others, that the way in which we can judge of the degree of their validity will be by means of the dialectic process. If the ideas are themselves stages in that process, the place which they occupy in it will give us their relative validity. If they are not stages in the process, their relative validity can be found by ascertaining the point in the dialectic at which it becomes clear that they are not absolutely valid. For example, as the dialectic passes away from Quantity, it becomes clear that no idea of pure mathematics can be absolutely valid.

For, whether those ideas are themselves categories or whether they are not, it is clear that their absolute validity would imply the absolute validity of the general conception of Quantity, as given in the dialectic. Thus, even if Hegel's judgments about mathematics were all wrong, that would not prevent his dialectic from being the foundation of right judgments on the same subject to a person more skilled in mathematics.

I.—(INDEFINITE) QUANTITY.

A.—Pure Quantity.

This stage (Greater Logic, 212; Encyclopædia, 99 [1]) is, as the Thesis of a new triad, identical in content with the last stage of Quality, but is expressed with greater immediacy. The last stage of Quality was the Relation of Attraction and Repulsion. In this, to recapitulate the conclusion of my last paper (Mind, 1902, p. 526), "The last trace of Quality has now died out. It had almost entirely gone when the Somethings had been transformed into Ones, each of which was exactly similar to all the others But a remnant still remained, in the shape of the Repulsion which each One exercised on all the rest. Now this Repulsion is swallowed up in a balance of Repulsion and Attraction. The Ones have now become indifferent to each other.

"And with this Quantity has been reached. Quantity involves that the units should be indifferent to one another—that they should be capable of combination or separation without any change in their nature. This is rendered possible by the indifference which has now been established. The Ones are sufficiently under the influence of Attraction to enable them to be brought together in aggregates. They are sufficiently under the influence of Repulsion to retain their separate existence in their aggregates, so that the quantity of the aggregate varies according to the number of its units.

"Quantity requires, also, that the units should be taken as equal to one another. And this condition, also, is satisfied by the Ones, which have no qualitative differentiations,

[1] My references in this paper to the Greater Logic are to the *pages* of vol. iii. of Hegel's *Works* (ed 1838); my references to the Encyclopædia are to *sections*. It is only the passages in the Greater Logic which I quote as supporting the view which I take. Those to the Encyclopædia give the passages in which the corresponding point is treated in the later work, whether the treatment be the same or different.

and are all exactly alike. At this point, therefore, the dialectic passes over into Quantity."

Pure Quantity then, being nothing but the general notion of Quantity, is identical with the last stage of Quality, except that we are now considering only the results gained, and not the process—the equilibrium of Repulsion and Attraction—by which we gained it. The two elements which up to this point—till Quantity has been reached—have been called Attraction and Repulsion are now, in Quantity, called Continuity and Discreteness. The only difference between Attraction and Repulsion on the one hand, and Continuity and Discreteness on the other, is that which is involved in the passage into Quantity—the perception of the fact that they are inseparable, that, as was demonstrated in the triad of Repulsion and Attraction (G. L., 190-200; Enc., 98), either is impossible without the other.

But, although they are recognised as inseparable, it is still possible to lay a greater emphasis on one of them than on the other. And we begin, Hegel tells us (G. L., 213), by laying a greater emphasis on Continuity. The reason for this is, mainly, that this element is more characteristic of Quantity, though not more essential to it, than Discreteness. For as long as we had only Repulsion the process remained within Quality, but, as soon as Attraction was added, the transition to Quantity took place. Apart from this, there is always a tendency to put most emphasis on the last gained element.

B.—Continuous and Discrete Magnitude.

By a somewhat abrupt transition we come to this category, in which Magnitude is to be taken first as Continuous (G. L., 229). A consequence of this is that there is yet no plurality of Quantities, and that the one Quantity is indefinite. For a plurality of Quantities would require that they should be Discrete from one another. And, again, no Quantity can be definite unless by its having fixed boundaries —that is to say, by being Discrete from the Quantity beyond those boundaries. It is true that, as was said above, all Quantity has an element of Discreteness. But, so far, the only things which are Discrete from one another are the units—the Ones—which are alike Discrete from and Continuous with one another.

Now a One, taken by itself, is not a Quantity at all. For it has no plurality in it. And Ones have no possibility of varying in magnitude. All variations of magnitude are only variations in the number of the Ones. These characteristics

are essential to Quantity, and they are not possessed by isolated Ones. And the isolated Ones being, so far, the only Discrete things, we have as yet no plurality of Quantities or definite Quantity.

(It may appear incorrect to say that a One admits of no plurality. Can we not, it may be asked, conceive an isolated One as consisting of two halves, four quarters, and so on? But when we do this we have passed to a higher conception—that of Discrete Magnitude, which will be the next conception to be reached. A One which consists of parts is no longer the *mere* One, which is all that the dialectic has got at present. It is something which, while from one point of view a unit, is, from another point of view, an aggregate of two or four units. And its nature forms therefore no contradiction to what we have said of the *mere* One which is all we have before us in Continuous Magnitude.

In the same way, we may, and always do, conceive the units of which an aggregate is made up, as having magnitude, and as being capable of having different magnitudes, and of varying in magnitude. But we can only do this in so far as we conceive each of them as made up in its turn of units, and so as not being *mere* units.)

We now pass to Discrete Magnitude (G. L., 229). Continuous Magnitude was formed by passing from One to One in virtue of their Continuity with each other. But each One is as really Discrete from all the others as it is Continuous with them. And this puts it in our power to stop at any One we like, and not to go on to the next. We can thus form a finite Quantity, beginning at any point and ending at any other point. And this Quantity, being cut off by its Discreteness from the indefinite Quantity beyond it, will be a finite Quantity. In the indefinite Quantity, again, other finite Quantities can be formed, and thus we get a plurality of finite Quantities.

In the form of this stage, as presented by Hegel, there appear to be two defects. The first is that no reason is given why we should pass from Pure Quantity to the new stage. The second is that, although Continuous and Discrete Magnitude is not divided into a subordinate triad, yet there is a distinct dialectic advance within it—namely from Continuous to Discrete Magnitude.

These defects seem to me to be merely a matter of arrangement. Continuous Magnitude is not really a fresh stage, or part of a fresh stage, at all. It is nothing but Pure Quantity, since, as we have seen, it does not permit of definite Quantity, or of a plurality of Quantities.

On the other hand, Discrete Magnitude is not merely correlative with Continuous Magnitude. It is distinctly a more advanced conception. It gives us the distinctness and plurality which were lacking before, and it gives them to us by differentiating the relation between Ones—by joining some of them to others, and disjoining them from others again, instead of making the relation uniform.

It is then, in reality, to Discrete Magnitude that the advance from Pure Quantity is made. This is evident in Hegel's text, but is misrepresented by his headings. In order that these should correspond with his argument, he should have dealt with Continuous Magnitude under the head of Pure Quantity, and should have made his second stage simply Discrete Magnitude, instead of Continuous and Discrete.

It should be remarked that, although the transition to Discrete Magnitude lies in the possibility of breaking off the Quantity at any One, this does not mean that it is merely a possible transition. Continuous Magnitude is that which cannot be broken off at any point. Discrete Magnitude is that which can be broken off at any point. When we are forced to admit the possibility of breaking Magnitude off at any point, this is a necessary transition to the category of Discrete Magnitude.

We can break it off, then, at any point we like. But no reason has been given why we should break it off at one point rather than another. Nor can any such reason be given until we have passed out of the sphere of Quantity into Measure. To this point we shall recur later on.

C.—Limitation of Quantity.

(G. L., 231.) Hegel says that Discrete Magnitude as such is not limited. It is only limited as separated from the Continuous. By this, I conceive, he means that, if the Discrete Magnitude were taken in isolation, its final One would not be a Limit, because it would not divide the Discrete Magnitude from anything else. It is only in so far as it is regarded as in connexion with the indefinite Continuous Magnitude from which it has been carved out, that its final term is to be considered a Limit. (On Hegel's use of Limit, cp. MIND, 1902, p. 513.)

The Discrete Magnitude thus shares its Limit with the Continuous Magnitude outside it. It is thus in a definite relation to that which bounds it, and has itself a definite amount. To definite Quantities Hegel gives the name of

Quanta, and so we pass to the second main division of our subject,

II.—QUANTUM.

A.—NUMBER.

(G. L., 232; Enc., 101.) In reaching the conception of a limited and definite Quantity we have reached for the first time the possibility of Number. While Quantity is merely Continuous it cannot be numbered. For then there is no intermediate term between the separate Ones and the whole unlimited indeterminate Quantity. The separate Ones in their separateness cannot have any Number, since each of them is only One. And, on the other hand, Indefinite Quantity can have no Number, since it has no Limit, and without a Limit it cannot have one Number rather than another, that is, it can have no Number at all. But now that we have a definite Quantum, it consists of those Ones which are included between certain limits, and can therefore be numbered.

"Quantity is Quantum," says Hegel, "or has a Limit, both as Continuous and as Discrete Magnitude. The difference of these species has here, to begin with, no meaning" (G. L., 232). This must not, of course, be taken as an assertion that Continuity and Discreteness have no longer meaning as different *moments* in any Quantity. It is only the distinction between Continuous and Discrete *Magnitudes* which has no longer any meaning. And we have seen that the two have been united in Limitation of Quantity. For there it became clear that Discrete Magnitudes, while, as their name shows, Discrete from each other, were also Continuous with each other. In fact, we may say that finite Quantities now stand to Quantity as a whole in the same relation in which Ones stand to finite Quantities—that is to say that they constitute it by virtue of being both Discrete from and Continuous with each other.

Quantity is now indifferent to its Limit, but not indifferent to having a Limit, for to have a Limit is identical with being a Quantum (G. L., 232). The distinction seems to be that it is always essential to a Quantity to have a Limit, but never essential to it to have a particular Limit. Of course, if it had a different Limit, it would be a different Quantity. But then there never is any reason why a Quantity should not be a different Quantity, unless non-quantitative considerations are introduced, which we have no right to do here.

This point will recur again when we come to the Quantitative Infinite Progress.

Hegel further says that the Ones which make up any Quantum are indifferent to the Limit, but that the Limit is not indifferent to the Ones (G. L., 234). As the Limit is that which determines the Quantum to be what it is, it follows that the Ones in a Quantum are indifferent to the Quantum, while the Quantum is not indifferent to them.

This superiority of the units to the aggregate is essential to Quantity, and is implied in all quantitative statements. When we say, for example, $7 = 5 + 2$, we assume that each of the units dealt with will remain unchanged, whether it is combined with more or fewer others. If not, then the proposition would not be true. But the aggregates do not remain the same, regardless of the units. If for example we take one unit away from 7, it is no longer equal to $5 + 2$.

B.—Extensive and Intensive Quantum.

(a) *Their Difference.*

Extensive and Intensive Quanta differ from one another in a manner analogous to the difference between Continuous and Discrete Quantity. The distinction between the new pair of terms and the old pair is that Extensive and Intensive refer to Quantitative *Limits* only, and, as the Quantum is identical with its Limit, they apply to Quanta, while, since no Quantities except Quanta have Limits, they apply to no Quantities except Quanta. Continuous and Discrete, on the other hand, are applicable to all Quantities (G. L., 252).

We first have Extensive Quantum. This conception is identical with that of Number, except that its determination is now explicitly posited as a plurality (*Vielheit*) (G. L., 253). I do not see why plurality is more explicitly posited in the conception of Extensive Quantum than in that of Number, nor does Hegel give any reason why it should be so. It can easily be understood, however, that the idea of Extensive Quantum has the same content with the idea of Number. The Extensive Quantum is looked on as primarily a plurality. It is not exclusively a plurality, for, since it is a Quantum, it must be definite, and, being definite, must be Discrete. It is therefore a unity as well as a plurality, but its distinctive mark is plurality. Now this is also the case with Number. A Number is a unity, or it could not be definite. But it is conceived as more essentially a plurality. This is clear from the atomism of Number mentioned above, by which the Ones are indifferent to the Quantum, but the Quantum is not

indifferent to them, which gives the plurality of the Ones a logical priority over the unity of Quantum.

But the plurality contained in each Number is not a plurality of unlike things, but of things with a similar nature, and Continuous with one other. They can therefore be taken as a unity, and, when this is done, we get the conception of Intensive Quantum (G. L., 253; Enc., 103).

The difference between Intensive and Extensive Quantum is one of comparative emphasis. Extensive Quantum has a certain unity, but it is subordinate to its plurality. Intensive Quantum has a certain plurality, but it is subordinate to its unity. The limit of an Intensive Quantum is called its Degree (G. L., 254; Enc., 103). The Degree of such a Quantum is a Majority rather than a Plurality (*Mehrheit* rather than *Mehreres*). And while it may be spoken of as a Number (*Zahl*), it must not be regarded as a Sum (*Anzahl*) (G. L., 254).

(*b*) *Identity of Extensive and Intensive Magnitude.*

The treatment of this point is rather obscure. Hegel says "Extensive and Intensive Magnitudes are thus one and the same determination of Quantum; they are only separated as follows, that one has its Sum inside itself, the other has its Sum outside itself. Extensive Magnitude passes over into Intensive Magnitude, since its plurality falls inherently into a unity, outside which plurality is found. But on the other hand this unity only finds its determination in a Sum, and in a Sum which is regarded as its own; as something which is indifferent to Intensities otherwise determined, it has the externality of the Sum in itself; and thus Intensive Magnitude is as essentially Extensive Magnitude" (G. L., 256).

Does this mean that the two terms are strictly correlative —that they stand side by side in the dialectic process, and that the transition from Intensive to Extensive is of precisely similar nature as the transition from Extensive to Intensive? Or does it mean that Intensive Quantum stands higher on the scale than Extensive, and that the transition from Extensive to Intensive is the transition of the dialectic process, while the transition from Intensive to Extensive merely means that what is seen under a higher category can, if we choose, also be regarded under a lower category?

The words quoted above suggest the first of these alternatives. And this is supported by the passage which immediately follows them (G. L., 257). In this we are told that with this identity we gain a Qualitative Something, since the identity is a unity which is formed by the negation

of its differences. This on the whole suggests that the two terms are to be taken as on an absolute equality.

Nevertheless, it seems to me that the weight of the evidence is on the whole in favour of the view which finds Intensive Magnitude a more advanced stage of the dialectic process than Extensive Magnitude. To this conclusion I am led by three reasons.

In the first place, we cannot safely lay much weight on Hegel's expressions about the Qualitative Something. For the introduction of a Qualitative element here seems merely casual. It is dropped as soon as it has been stated. We hear nothing more of it while we remain in the division of Quantum. The next mention of a Qualitative element comes in the division which succeeds Quantum—namely Quantitative Relation. And when it comes in there, it is introduced quite independently, with no reference to the passage on page 257, and in quite a different way. That passage cannot therefore be considered one of much significance.

In the second place, the transition to the next category (The Alteration of Quantum) does not start from the identity of Extensive and Intensive Magnitudes, but from the conception of Intensive Magnitude taken by itself. This will, I think, be evident when we come to consider the transition, and it would follow that Intensive Magnitude must be above Extensive in the scale of categories, since the movement to further categories passes from the Intensive, taking no special account of the Extensive, which must therefore be considered as absorbed in the Intensive.

In the third place, this view is supported by several passages of Hegel. He says (G. L., 279-280) that the notion of Quantum reaches its reality as Intensive Magnitude, and is now posited in its determinate Being as it is in its Notion. This is supported by the Encyclopædia, where he says (Enc., 104) that in Degree the notion of Quantum is explicitly put. There is also not the slightest doubt that, in the Encyclopædia, Intensive Quantum is higher than Extensive Quantum, for, under the name of Degree, it forms a quite separate division, which is the last and highest division of the whole of Quantity.

On the whole, therefore, although the evidence is certainly conflicting, I think it better to hold that Hegel regards Intensive Quantum as higher than Extensive Quantum. We can easily see why it should be regarded as higher. It emphasises the unity of the Quantum rather than its plurality. In other words, it emphasises the Limit. Now this emphasis of the unity and the Limit carries us farther

away from the indefinite Quantity with which Quantity as a whole began. In that indefinite Quantity there were no Quantities, each with a Limit and unity of its own. Thus the more emphasis is laid on unity, the farther do we get from the previous stage, and this is an advance. And, again, the more the unity of each Quantum is recognised, the more pressing becomes the question why it should be that Quantum, and not some other—the question which will carry us over into the last subdivision of Quantum, which is Quantitative Infinity.

Once again, then, Hegel's titles do injustice to the course of his argument. The real advance is not from the difference between Extensive Quantum and Intensive Quantum to the identity between them. It is rather from Extensive Quantum to Intensive Quantum. And thus it would seem that the two first subdivisions of Quantum should have been (*a*) Extensive Quantum, (*b*) Intensive Quantum.

We have come thus, for the second time in this paper, to the conclusion that Hegel's titles do not do justice to the merits of his argument. In each case the defect arose from the titles taking as correlative two conceptions, of which his argument shows one to be superior to the other. In the first case it was the Continuous and Discrete; in the second case it was the Extensive and Intensive. It may perhaps be the case that the confusion arose from following in the titles the usage of mathematics, for which each of these pairs is a pair of two correlatives which are on a strict equality with one another. Should this be the true explanation, it would add another to the cases in which the consideration of the finite sciences, so far from rendering assistance to the dialectic, has distorted it, and injured its cogency. Such, as I have endeavoured to show in former papers, was the case with the categories of Chemism and Life.

We now come to the transition to the next category. Of this Hegel says: "The Quantum is the determination posited as transcended, the indifferent limit, the determination which is equally the negation of itself. This discrepancy is developed in Extensive Magnitude, but it is Intensive Magnitude, which is the determinate being of this externality, which constitutes the intrinsic nature of the Quantum. It is posited as its own contradiction, as being the simple determination relating itself to itself, which is the negation of itself, as not having its determination in itself, but in another Quantum.

"A Quantum is therefore posited as in absolute Continuity, in respect of its Quality with what is external to it, with its

Other. It is therefore not only *possible* that it should go beyond any determination of Magnitude, it is not only *possible* that it should be altered, but it is posited as *necessarily* altering itself. The determination of Magnitude continues itself in its Otherbeing in such a way that it has its being only in its Continuity with an Other; it is a limit which *is* not, but *becomes*" (G. L., 261; cp. also Enc., 104).

In other words, a Quantum can only be defined in relation to another Quantum. No reason can ever be found in any Quantum (if non-quantitative considerations are eliminated) why it should have its actual Magnitude rather than some other. All Magnitudes are fixed by non-quantitative considerations. There is an *a priori* reason why a triangle has three sides, rather than two or four. There is an empirical reason why there are seven apples on this dish, rather than six or eight. But there can never be any reason why the number seven, taken simply as a number, should not be in any particular case replaced by six or eight. It has its determination in another Quantity — it stops where another begins. But it is after all continuous with this other Quantity —the Ones are just the same on each side of the Limit, and there can be no reason why the Limit should not be put elsewhere, and so add to the Quantum or diminish it. And so we come to

(c) *The Alteration of Quantum.*

Why, it may be asked, did not this conception of the necessary variation of Quantity come before? Surely it is as true of an Extensive Quantum as of an Intensive Quantum that no reason can be found in the nature of the Quantum itself why it should not be larger or smaller.

I think it is true that, if we had stopped at Extensive Quantum, without going on to Intensive, this conception of Alteration would have necessarily followed from Extensive Quantum. But the more immediately obvious transition—and therefore the one to take first—was the transition to Intensive Quantum. And, if Intensive Quantum was to come in at all, the transition to Alteration of Quantum comes better after it, for the necessity of that transition then becomes far more obvious. As was said in the passage quoted above, it was developed in Extensive Magnitude, but finds its determinate being in Intensive Magnitude.

When we regard a Quantum as Extensive, we regard the plurality of Ones as the element which is logically prior, and the Quantum as a whole is regarded as dependent on

the Ones. Now so long as we refer the Quantum to the Ones, there *is* a reason for the Quantum being the size it is, and no other—namely that it includes those Ones, and no others. If we go farther, and ask why those Ones and no others should be included, no answer could be given, and the conception of Alteration would arise, but so long as we regard the Ones as ultimate in reference to the Quantum, the necessity of Alteration remains in the background.

But with Intensive Quantum it comes at once to the front. For there the unity of the Quantum is the prominent element. It is conceived as logically prior to the Ones. And therefore our question—why is it this Quantum, and not a larger or smaller one—cannot be referred back to the Ones which it contains. And therefore the necessity of Alteration, which is due to the impossibility of answering this question, follows more obviously and naturally from Intensive Quantum.

This is what Hegel means when he says (G. L., 253) that determination of a Quantum through Number (which is a category previous to Intensive Quantum) does not need another Magnitude, because in Number Quantum has its externality, and its relation to another, inside itself. (If this passage seems to deny *all* tendency to Alteration or the fact of an Extensive Quantum, we must remember the explicit assertion on page 261 that the difference in this respect between Extensive and Intensive is merely a matter of degree.) And again (G. L., 254) "Degree, therefore, which is simple and in itself, and so has its external Otherbeing no longer in itself, has that Otherbeing outside itself, and relates itself to it as to its determination".

We have now come to the end of Extensive and Intensive Quantum, and pass on to the third subdivision of Quantum, which is called

C.—The Quantitative Infinity.

(*a*) *Its Notion.*

The first subdivision of Quantitative Infinity is, as usual, the restatement of the last subdivision of the preceding triad. The first movement of the Quantum when it passes its Limit is into a Quantity which is simply defined as not being that Quantum. So far, then, it is only Quantity, and no longer Quantum. And as Quantity is only bounded when it is Quantum, this Quantity has no boundaries at all. Thus it is infinite (G. L., 263).

Hegel now proceeds to remark on the difference between the Qualitative Infinity, which was one of the triads in

Being Determinate, and the Quantitative Infinity, with which we are now dealing (G. L., 264). That which is Qualitatively determined is not *posited* as having the other in itself. Magnitudes, on the other hand, are posited as being essentially Alterable, as being, in Hegel's somewhat peculiar language, "unequal to themselves and indifferent to themselves".

The difference is one which always arises between lower and higher categories in Hegel's philosophy. The method of the dialectic changes gradually as the dialectic process advances (cp. Enc., 240; 111, lecture note; 161, lecture note). It becomes more of a spontaneous advance from category to category, and less of a breaking down, by negative methods, of the resistance of categories which oppose any movement beyond them. It is thus to be expected, since Quantity comes later than Quality in the process, that the finite in Quantity should lead on to the infinite more expressly and directly than the finite in Quality does.

From this category the transition to the Infinite Progress takes place in a manner analogous to that which we noticed when we dealt with the Qualitative Infinite (MIND, 1902, p. 517). The Quantum is after all continuous with the indefinite Quantity into which it has passed over. If it were not, it would not have passed over into it. The passage has only taken place because both terms are Quantities, only separated by a Limit to which it is the nature of Quantity to be indifferent. But the Quantity on the other side of the Limit will also be composed of Ones, and thus the argument is again applicable which originally transformed Quantity into Quantum. The Other Side (*Jenseits*) of the original Quantum is now itself a Quantum. And therefore it, like the original Quantum, is essentially subject to alteration, and will pass the Limit, only thereby to reach a third Quantum, which will be surpassed in its turn, and so on (G. L., 265). Thus we come to

(*b*) *The Quantitative Infinite Progress.*

At this point Hegel inserts an interesting note on the supposed sublimity of the sort of Infinite which is revealed in such a progress as this. Such an Infinite, he says, can produce nothing but weariness (G. L., 268; Enc., 104, lecture note). This is extremely characteristic of Hegel. When he says that the true Infinite is not the unbounded, but the self-determined, he does not merely change the meaning of a word, but claims for the self-determined all

the dignity which is more commonly attributed to the unbounded. It is, perhaps, to his deep conviction that true greatness lies in self-limitation, and not in the absence of limitation, that we are to ascribe much of the special reverence which he shows for the ideas of the Greeks, as well as his contempt for the Romanticism of his own age and country.

At the same time we must not forget that Hegel never says that the False Infinite of an Infinite Series is necessarily contradictory, though he does say it is worthless and tedious. (Cp. MIND, 1902, p. 518: "The contradiction only arises when, on the one hand, it is asserted that something is explicable or determinable, and when, on the other hand, the attempt to explain or determine it leads to an infinite series. For we cannot tell that the series will be infinite, unless we know that *no* term in the series can give the required explanation or determination. And, if no term can give it, and the explanation or determination can only be looked for in the series, then it will not be found at all, which contradicts the original assertion that it can be found

"In opposition to this, it may perhaps be said that, though no term can give the required explanation or determination, the whole series may. But if the series is a mere aggregate of its terms, it can give nothing that is not given by one of them. And if the series is something more than the mere aggregate of its terms, then the solution is found in its unity, and not in the infinite series at all.")

Now it *is* an attempt to determine something which leads, in the case before us, to the Infinite Series. The dialectic process had reached the idea of a Quantum, which, among other characteristics, had to be definite. But it could only be definite by having a Limit, and keeping within it. We have seen, however, that any Quantum necessarily passes its Limit, and overflows into a fresh Quantum. But it is of the essence of Quantum to be determined, and the dialectic process will not permit us to reject the idea of Quantum altogether. In this case, therefore, a contradiction arises.

How is the contradiction to be avoided? In a very similar way to that in which the same difficulty was met in the case of Qualitative Infinity. That which is outside any Quantum is another Quantum. If we try to find the determination of any Quantum in itself exclusively, then we find that its Limit continually alters, and that the task is endless. But, if we fully accept the relation of each Quantum to the other which is outside it, the case is changed. No Quantum can determine itself as against another Quantum. But two

Quanta can reciprocally determine one another. There is no reason why 7 should not become 6, or why 17 should not become 16, if we take 7 and 17 as two isolated facts, each of which must be determined by itself, or not at all. But if we take these Quanta as related to one another, then there is a reason why 7 should not become 6—for it would then bear a different relation to 17, and there is a reason why 17 should not become 16—for it would then bear a different relation to 7. Thus the Quanta have now some real self-determination, though it is slight; *a* cannot become greater or less, because it would thereby change its relation to *b*. And its relation to *b* is what it is, not only because *b* is *b*, but because *a* is *a*. With this partial self-determination we reach (G. L., 279; Enc., 105, lecture note)

(*c*) *The Infinity of Quantum*,

by which is meant the true Infinity of self-determination, as opposed to the False Infinity of an unending progress.

It will be noticed that there is a difference between the Quantitative Infinite Progress and the earlier Qualitative Infinite Progress. In Quality (cp. MIND, 1902, p. 517) the Something finds its nature only in another Something, which in turn finds its nature in a third, and so on. The Somethings themselves do not change, but fresh ones are continually reached, in the vain search for a final determination. In Quantity, however, the Infinite Progress is not one of an Infinity of Quanta, but of a single Quantum, which endlessly increases in size, as it successively overleaps every Limit.

This difference is inevitable. In Quality there can be no change of anything. The nature of reality is not yet sufficiently complex to allow anything to become different in one respect while remaining the same in others. If a thing is not completely the same it has utterly vanished (cp. MIND, 1902, p. 508). It is impossible, therefore, for a Something to change, and the Infinite Progress can only proceed by adding fresh Somethings.

In Quantity the position is altered. Change is now possible, and so the original Quantum can change. On the other hand, the indifference of the Quantum to its Limit (the first correction of which only arises as we pass out of the Quantitative Infinite Progress) renders it impossible to pass from one Quantum to another.

This difference of the Antitheses in the two triads accounts for the difference in the Syntheses, though the general thought in both Syntheses is the same.

With this stage of the dialectic the idea of Quality returns (G. L., 281; Enc., 105). This is most clearly stated in the Encyclopædia: "That the Quantum in its independent character is external to itself is what constitutes its quality. In that externality it is itself and referred connectively to itself. There is a union in it of externality, *i.e.*, the quantitative, and of independency (Being-for-self)—the qualitative". The essential characteristic of Quantity was that it could alter and yet remain the same. Now this characteristic begins to disappear. The Quantum can no longer alter without the least effect on anything but its own Magnitude. For it is now in relation to some other Quantum, and it cannot alter unless either that other Quantum, or the relation, alters simultaneously. This is the first step (though as yet but a very small one) towards bringing back, on a higher level, the fixity of Quality. With it we pass out of Quantum, to the third and last division of our subject, after some mathematical digressions occupying nearly 100 pages,

III.—THE QUANTITATIVE RATIO.

The Ratio between two Quanta is, as Hegel points out, itself a Quantum (G. L., 380). And he now transfers his attention from the related Quanta to the Quantum which forms their relation, and is known as the Exponent. If he can transcend the essential defect of Quantity in this case, he will have found a universal solution, since it is obvious that any Quantum can be expressed as the Ratio between two other Quanta.

The first and simplest form of Ratio is called

A.—THE DIRECT RATIO

(G. L., 381), which is a restatement of the last subdivision of Quantitative Infinity. The related Quanta are here taken as logically prior, and the Quantum which is their Ratio as logically subsequent. Thus we get, for example, that the Ratio of 7 to 35 is 5.

Hegel points out three characteristics of this Ratio. The first is that the Quantum which is the Ratio is no more determined by the two Quanta of which it is the Ratio than it is by an infinite number of pairs of other Quanta. For example, 5 is equally the Ratio of 6 and 30, of 8 and 40, and so on (G. L., 382).

The second characteristic follows from the first. The related Quanta cease, so far as they are taken simply in

this relation, to be perfect Quanta. For it does not matter how much they alter absolutely, provided they do not alter relatively. So long as one remains five times the other, they may both increase or decrease indefinitely. And the alteration of each is no longer perfectly free, but is conditional on an alteration of the other. If 7 increases to 9, then 35 must increase to 45 (G. L., 382, 383).

The third characteristic is that the whole meaning of the pair of related Quanta, taken as related, is summed up in the Exponent. And therefore Hegel finds it a defect in this category that the Exponent is not sufficiently marked out from the other Quanta. It cannot be the largest of the three Quanta concerned, but it can be either of the others. We have said that 7 and 35 stand to each other in a Ratio expressed by 5. But we might just as well have said that 5 and 35 stand to each other in a Ratio expressed by 7 (G. L., 383). Since—this appears to be Hegel's argument—the Exponent is specifically different from the related Quanta, it must be clearly distinguishable from them. But in Direct Ratio this is not the case. We must therefore seek another Ratio, where the Exponent is marked out by the nature of the relation. Now, if you have three integral numbers (and Hegel appears to assume that all his Ratios will be between Quanta expressed by integral numbers), there is a relation between them which has the required definiteness. If one of them is the product of the other two, then it is the largest of the three that will be the product. So we come to

B.—The Inverse Ratio

(G. L., 384), where the Exponent is the product of the two related Quanta. It appears to be called Inverse because the increase of one of the related Quanta involves the diminution of the other.

The transition to the next category is extremely obscure. So far as I can understand it, it is as follows (G. L., 389). Either of the two related Quanta can increase, so long as the other diminishes, the only Limit of this process being that neither of the related Quanta can become larger than the Exponent. Thus either of the related Quanta is implicitly (*an sich*) the Exponent. Hegel calls this "the negation of the externality of the Exponent". This means, if I am correct, that there are no longer necessarily *three* Quanta, but only two, namely the Exponent, connected with one other Quantum, no longer by a third Quantum, but by some non-quantitative relation. And thus, says Hegel, without any further explanation, we reach

C.—The Ratio of Powers.

By this he appears to mean only the special relation which exists between two numbers, one of which is the square of the others (G. L., 390). It is the square, as the result of the process, which is treated as the Exponent.

The transition appears very questionable. It may be admitted that the indefinite approximation of one of the related Quanta to the Exponent brings a Qualitative element into greater prominence, and that the Ratio of Powers has also a relatively prominent Qualitative element. But in other respects they are quite different conceptions. And Hegel gives us no reason for passing at this point from one partially-qualitative relation to another and distinct partially-qualitative relation. He is satisfied with showing that they are both partially-qualitative, which is clearly not sufficient.

It is difficult to see, too, why Hegel thought himself justified in considering only those cases where one Quantum was the square of the other, and in excluding cubes and other powers. If, however, he *had* considered these other powers, it would have become evident that the relation between the two Quanta was not yet one which could dispense with a third Quantum. For the question of the power to which one was to be raised to equal the other could only be answered by naming a third Quantum.

Hegel makes the transition to the next category as follows: "Quantity as such appears as opposed to Quality; but Quantity is itself *a* Quality, a determination in general which relates itself to itself, separated from the determination which is other than it, from Quality as such. Yet it is not only *a* Quality, but the truth of Quality itself is Quantity; Quality has shown itself as going over into Quantity; Quantity, on the other hand, is in its truth that externality which is turned back on itself, which is not indifferent. So it is Quality itself, in such a way that outside this determination Quality as such is no longer anything" (G. L., 392). He goes on to say that this union of Quantity and Quality gives us Measure, which carries us beyond our present subject into the third and last subdivision of the Doctrine of Being.

We have now reached the end of Hegel's treatment of Quantitative Ratio. Can it be regarded as valid? I do not think that it can. Something might perhaps be said against the validity of the transition from Direct to Inverse Ratio. Certainly a good deal might be said, as I suggested above, against the transition from Inverse Ratio to the Ratio of Powers. But it is not necessary to go into these difficulties,

for there is a much more general objection. The whole triad of Quantitative Ratio is a blind alley. It does not lead, as it professes to lead, to the category of Measure, and the chain of the dialectic cannot be continued through it.

The passage I have quoted above contains the transition from Quantity to Measure. We therefore have before us the manner in which the inadequacies of Quantity are to be transcended, and in which Quality is to be recovered and synthesised with Quantity in Measure. It seems to me that neither of these objects has been really attained.

As to the first. The special characteristic of Quantity was its indifference. It was originally stated to be that which could alter, and yet remain the same. When we reached Quantitative Infinity, we found that it not only could alter, but must alter, and it was to remedy the contradictions thus caused that we were forced to have recourse to Quantitative Ratio.

Does Quantitative Ratio remove this indifference, even when taken in its highest form, the Ratio of Powers? Let us pass over the difficulty that the power to which a number is to be raised can only be expressed as an immediate Quantum, which might be any other. Let us confine ourselves, as Hegel does, to squares, and ignore the quantitative nature of the index. Has this removed the indifference? If we take 49 as a simple Quantum, it is under the necessity of changing continually. If we take it as the square of 7, has the necessity disappeared?

Surely it has not. It is true that 49 cannot now change unless the 7 changes with it. But 7 is also a Quantum, and so there can be no reason why it should not change, nor, therefore, why 49 should not change. Again, the first numbers it can change to are no longer 48 and 50, but 36 and 64. But its number of changes is still unlimited, since any number may have a square. There is no end to the various numbers which can be substituted for 7, and, therefore, no end to the various numbers which can be substituted for 49. The movement of the 49 has now a few restrictions put upon it, but not sufficient to save it from the possibility and necessity of continuing in an infinite series. And therefore Quantitative Ratio has not removed the contradictions of Quantitative Infinity, nor has it enabled us to transcend the characteristic nature of Quantity. It is true that 7 and 49 are linked Quanta, but they are still Quanta.

With this is very closely connected the second defect of the triad. It professes to lead us to Measure, and it must therefore bring back Quality. In the passage quoted above

(G. L., 392) Hegel says that it has done this. We may admit the first part of what he says. Since the conception of related Quanta was first introduced in the category of Infinity of Quantum, there has been a slight Qualitative element in the nature of Quantity. For the movements of each separate Quantum are no longer completely arbitrary and unconditioned, and every restriction on the movement means some departure from the typical idea of Quantity. But this is not enough. In the Ratio of Powers we have the transition to Measure. In it, therefore, Quality ought to be completely restored, since Measure is the Synthesis of Quality and Quantity. It ought to be present as something which is indeed united with Quantity in the Synthesis, but which is no more dependent on, or a variety of Quantity, than Quantity is a variety of it. This has not happened. We have got a Quantity, which is more like a Quality than before, but which is still essentially a Quantity, and not a Quality. The test of this is the indifference, and the Infinite Progress which the indifference gives rise to. Till we have got rid of this, we have not transcended Quantity. For the indifference is, as we have seen, the special characteristic of Quantity, and it is also the source of the contradictions inherent in Quantity, for the removal of which the transition to Measure becomes necessary. But, as I pointed out above, the Ratio of Powers does not get rid of the indifference or of the Infinite Progress. For it can only account for the size of one Quantity by its relation to another. And if we ask why the other is no larger or smaller, we can only be referred to a relation which it had with some other Quantum, and so on continually. Our conclusion must be that the Ratio of Powers has not transcended Quantity, and is not, therefore, a valid transition to Measure.

What then is to be done? We saw reason to think that the transition from Quantum to Quantitative Ratio is valid, and I believe that it is possible to recast the triad of Quantitative Ratio in such a way as to make a valid transition to Measure. The Thesis of my proposed triad would be the restatement of the general idea of Quantitative Ratio, as it had been arrived at in the previous category of Infinity of Quantum. It might be called *Quantitative Ratio as such*, or again *Quantitative Ratio in general* (*überhaupt*), either of which would be in accordance with Hegel's terminology.

The inadequacy of the Thesis would lie in the fact, which we have already mentioned, that, if one Quantum is determined by its Ratio to another, the question inevitably arises how that other is to be determined. We are thus led into an

infinite series. This conception forms the Antithesis of our triad, and might be called *The Infinite Series of Ratios*.

It will be noticed that this Infinite Series resembles the Infinite Progress found in Quality more than it resembles the Infinite Progress in Quantum. For the Ratios do not continually alter, as the Quanta did. The Infinity comes in through the necessity of going on to fresh Ratios to determine those already existing. This approximation to the Qualitative type of infinity is very natural, since, with Ratio, Quantity has begun to approximate to Quality.

Here, as in the two previous cases, the Infinite Series involves a contradiction. The original Quantum is determined. But it can only be determined by its relation to the next, and so cannot be determined unless that one is determined also. But this depends in like manner on the next again, and so on. Therefore the Original Quanta cannot be determined until an infinite series is completed. That is, it can never be determined, which contradicts the previous assertion that it is determined.

We must pass on, then, to a fresh category, which will remove this contradiction, and will form the Synthesis of Quantitative Ratio. We have seen that Quantity, however developed, can never, while it remains only Quantity, get rid of the inadequacy which has now shown itself once more in the Infinite Series of Ratios. Now the ground of this inadequacy was the necessary indifference of all Quanta. And this indifference, we saw, proceeded from the fact that all Ones were exactly alike, so that there could be no reason assigned why a Quantum should stop at any particular Limit, rather than any other.

The only way of escaping from our difficulty, therefore, will be to reject the exact similarity of the Ones. At the same time, we must not reject all that has been gained since Being Determinate was left behind. For, if we did replace ourselves in the position of Being Determinate, then all the categories would again be developed from it till Quantitative Ratio was reached, when we should have again to return to Being Determinate, and so on in an endless round.

It is necessary, then, to keep Quantity in some form, and yet to restore Qualitative differences. Now this can be done, if at certain points in a series of units there is a Qualitative change, so that the Ones on one side of each of these points are Qualitatively different from those on the other side of that point. In this way we shall still have Quanta, because we shall have, within certain limits, aggregates of Ones which are of precisely similar nature. And since, at

these limits, there is a Qualitative change, there is now a reason why the Quantum should remain within its Limit, and not increase beyond it. For it is a Quantum of Bs, and, if it went beyond the Limit, it would find no more Bs but only Cs.

We have thus reached a solution of the inadequacy of Quantitative Ratio, and also of the inadequacy of Quantity generally. The conception which has achieved this is identical with the category to which Hegel gives the name of Measure. The third member, therefore, of the triad of Quantitative Ratio may, in accordance with Hegel's terminology, be called *The Transition to Measure*. And with this we pass from Quantity to Measure—the third and last subdivision of the Doctrine of Being.

The course of the argument in the Encyclopædia is practically the same as in the Greater Logic, except in the relative importance given to different categories. In the Greater Logic, as we have seen, Extensive and Intensive Magnitudes, and the Infinite Progress, all fall within the second subdivision, while the third subdivision is completely taken up by Ratio. In the Encyclopædia, the second subdivision (named, as in the Greater Logic, Quantum) deals with Extensive Magnitude only. The third subdivision is called Degree, and contains Intensive Magnitude, the Infinite Progress, and Ratio. This arrangement shows more clearly the advance made in passing from Extensive to Intensive Magnitude, but otherwise it seems inferior to the order of the Greater Logic. For Intensive Magnitude seems more closely connected with Extensive Magnitude than it is with Ratio. And, again, the Infinite Progress makes manifest the characteristic contradiction inherent in all Quantity. It would seem, therefore, more appropriately placed in the second subdivision, which is the Antithesis of the triad of Quantity, than in the third, which is the Synthesis.